

**Due Date:** Wednesday September 23, 2020, 9:00pm

## Instructions

Answer all questions on paper or on a tablet using your own handwriting. **Please number each page.** Include a cover page with your name, student ID and a list of the questions you have answered. The TA will record your mark on the cover page. If you use paper make a photo of each page or scan each page and upload your solutions to crowdmark.

### Textbook Reading

- Sections: 1.1, 1.2, 1.3, 1.4

### Definitions, Concepts & Keywords

- Know the difference between strings, sets, subsets and permutations.
- Understand the basic terminology on graphs and trees.
- Know how to count permutations, combinations, and use the Binomial Theorem.
- Know how to count permutations and combinations with repetition.

### Exercises

#### A. Textbook Questions : Grimaldi, 5th edition

Section 1.2 Exercises 5, 20.

Section 1.3 Exercises 6, 30.

Section 1.4 Exercises 1, 10.

#### B. Instructors Questions

1. DNA sequences have an alphabet  $\{A, C, G, T\}$ .  
How many DNA sequences of length  $n$  are there?
2. List all binary strings of length 5 with three 1 bits.  
You should get  $\binom{5}{3}$  of them.
3. Draw a graph  $G$  with 6 vertices where each vertex has degree 2.  
Draw a different graph  $H$  with 6 vertices where each vertex has degree 2.
4. Draw all unlabelled trees with 5 vertices.
5. Let  $G$  be a graph with vertex set  $V = \{1, 2, 3, 4, 5, 6\}$ . Suppose  $G$  has no edges between odd numbered vertices and also no edges between even numbered vertices.
  - (a) Draw such a graph with the maximum number of edges.
  - (b) How many such graphs are there?
6. (a) How many *combinations* of size 3 can one produce from  $a, b, c, d, e, f$ ?  
(b) How many *permutations* of size 3 can one produce from  $a, b, c, d, e, f$ ?
7. How many cycles of length 4 edges are in the complete graph  $K_5$ ?  
Note: the cycle 1-2-3-4-1 is the same cycle as 2-3-4-1-2.
8. Let  $A = \{a, b, c, d\}$  and  $B = \{1, 2, 3, 4\}$ . Let  $f(x)$  be a function from  $A$  to  $B$ .  
One such function is shown in the table below.

$x$	$a$	$b$	$c$	$d$
$f(x)$	1	3	1	2

- (a) Find the number of functions from  $A$  to  $B$ .
- (b) Find the number of onto functions from  $A$  to  $B$ .

9. Six people are sitting at a circular table.  
How many (circular) permutations of them are there?  
Note, the circular permutations 123456 and 234561 are considered the same.
10. Write out the proof that  $\binom{n}{k} = \frac{n!}{k!(n-k)!}$  from the class notes.
11. If  $\binom{30}{20} = 30,045,015$  what is  $\binom{30}{10}$ ?
12. Show that  $\binom{n}{2}\binom{n-2}{2} = 6\binom{n}{4}$ .
13. A box has 10 red balls, 10 green balls, 10 blue balls and 10 yellow balls. Each set of 10 balls is numbered 1 to 10.  
Suppose four balls are chosen from the box.
  - (a) How many ways can there be four of the same colour?
  - (b) How many ways can there be three of one colour and one of another colour?
  - (c) How many ways can there be two of one colour and two of a different colour.
14. Determine the coefficient of
  - (a)  $x^2y^3z^2$  in  $(x+y+z)^7$
  - (b)  $x^2y^3z^2$  in  $(x+2y+z)^7$
15. How many ways can we distribute 10 apples to 4 people such that each person gets at least one apple?
16. How many ways are there to distribute 5 muffins and 4 bagels to 3 people?