MACM 202 Assignment 6, Fall 2005

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This assignment is worth 10% of your grade. It is due Tuesday December 6th at 2pm. A late penalty of 20% will apply for each day late. Attempt all questions.

Question 1: Linear First Order Systems (25 marks)

Exercises 5.6 and 5.10 from the text.

For problem 5.6 use Maple for parts (c) and (d).

Question 2: The House Warming Problem (15 marks)

Suppose we have a house with four rooms A, B, C and D arranged in a square with a furnace F in room A. To simplify the problem you may assume that the house is symmetric.

Let $T_A(t), T_B(t), T_C(t)$ and $T_D(t)$ be the temperature in rooms A, B, C and D at time t and let the outside temperature be T_O.

(a) Using Newton's law of cooling, write down a system of four first order differential equations for $T'_A(t), T'_B(t), T'_C(t)$ and $T'_D(t)$ which model the change in temperature of the four rooms.

(b) Determine the equilibrium temperature as a function of T_O, F and the other constants using the `solve` command. To simplify the formulae, collect them in T_O and F using the `collect` command.

(c) For $T_O = 0, T_A(0) = 0, T_B(0) = 0, T_C(0) = 0, T_D(0) = 0$ and suitable values of F and the other parameters, generate a plot of $T_A(t), T_B(t), T_C(t), T_D(t)$ on the same plot.

Question 3: Non-Linear First Order Systems (25 marks)

Exercises 5.16 and 5.17 from the text.

For 5.16 part (c) use DEplot. For 5.16 part (e), give a physical interpretation for each term (or pair of terms) in the two differential equations.

For 5.17 part (d) graph each type of behaviour found using DEplot.

Question 4: A Spring Modeling Problem (15 marks)

This problem is to model one of the vibrations of CO_2, carbon dioxide. Let M_C be the mass of a carbon atom and M_O the mass of an oxygen atom. We will model a vibration of the bonds connecting the carbon atom to the oxygen atoms by connecting them with springs. Place the atoms on the x axis with the carbon atom C in the middle. Connect each oxygen atom to the carbon atom by a spring with spring constant k. Let d be the rest length of the spring, i.e., if the carbon atom is distance d from the oxygen atom, no force is applied by the spring to either atom.

Let $c(t)$ be the position of the carbon atom at time t and let $y(t)$ and $z(t)$ be the positions of the oxygen atoms at time t. Assume the rest position of the atoms are at $x = -d, 0, d$ for the oxygen, carbon and oxygen atoms respectively.

Assuming Hooke's law, construct a system of three second order differential equations to model the motion of the three atoms assuming no friction. Let $v_c(t) = x'(t)$, $v_y(t) = y'(t)$ and $v_z(t) = z'(t)$. Now write the system as a system of six first order differential equations.

For suitable values for M_C, M_O, k, d, and

$$d = 2, c(0) = 0, c'(0) = 0, y(0) = -1.5d, y'(0) = 0, z(0) = 1.5d, z'(0) = 0$$

solve the differential equations using dsolve and graph the solutions. You should obtain $C(t) = 0$ and an oscillating solution for $y(t)$ and $z(t)$ if the differential equations are correct. Now, for

$$d = 2, c(0) = 0, c'(0) = 0, y(0) = -d, y'(0) = 0, z(0) = 1.5d, z'(0) = 0$$

solve the equations using dsolve and graph the solutions.

As a bonus for 10 marks, generate an animation of the three atoms placed on the horizontal axis. Represent each atom as a square using POLYGONS($[p, q, r, s]$, COLOR(...)). Represent the i'th frame of the animation as PLOT($O1_i$, C_i, $O2_i$) where $O1_i, O2_i, C_i$ are POLYGONs. Show me the animation.

Question 5: Partial Derivative Plots (20 marks)

Consider the function $f(x, y) = 1 - x^2 - y^3 - 2xy$.

(a) Generate a 3-dimensional plot of $f(x, y)$ on $-2 < x < 2$ and $-2 < y < 2$ depicting also the lines $f(x, 1)$ and $f(1, y)$. See the `plots[spacecurve]` command for drawing a line in 3-dimensions.

When drawing the surface $f(x, y)$ using the `plot3d` command, use the options `axes=frame` and `style=patchcontour`. To make the curves $f(x, 1)$ and $f(1, y)$ more visible, use the `thickness=n` option.

(b) To graphically depict the partial derivatives of f at the point $(x = 1, y = 1)$, include in your plot from part (a) the tangent lines at $(x = 1, y = 1)$.

(c) Graph the tangent plane at $(x = 1, y = 1)$ together with your plot from part (a).

(d) Compute the critical points of $f(x, y)$, i.e., the solutions of $\{f_x(x, y) = 0, f_y(x, y) = 0\}$ using the `solve` command. Generate a 3-dimensional plot of $f(x, y)$ which shows visually the location of the critical points by drawing a vertical line segment through each critical point (see the `plottools` package). Visually identify whether each critical point is a local minimum, local maximum, saddle point or inflexion point.