

Assignment 1, MACM 204, Fall 2022

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The purpose of this assignment is to learn the basics of Maple: how to input formulas, how to graph functions, solve equations, calculate derivatives and integrals, and to program some loops.

Due 11pm Monday September 26th. Late penalty. -20% for up to 24 hours late. 0 after that.

Do all calculations in Maple. Please put your name and student number on top of each Maple worksheet, export your Maple worksheets to .pdf files and upload them to Crowdmark.

Question 1

Consider the function $f(x) = x^2 e^{-x}$. Construct a plot of $[f(x), f'(x), f''(x)]$ on the same graph for the domain $0 \leq x \leq 6$.

Using an option, colour the three functions red, green and blue and include a suitable title. Read the help page for `?plot,options`.

Solution 1

Question 2

Using the `evalf` command, calculate the following values **to 20 decimal places**.

$\pi = 3.141\dots$, $\sqrt{2} = 1.414\dots$, $e = 2.718\dots$, $\int_0^1 e^{-x^2} dx = 0.746\dots$ and $f'(1) = 3.42\dots$ where $f(x) = \tan(x)$.

Solution 2

Question 3

Consider the polynomials $f := x^4 - 1$ and $g := x^4 - 4x^3 + 8x - 4$ and

$h := x^4 - 4x^3 + 4x^2 - 8x + 4$.

Graph each separately on a suitable domain for x so that we can see all real roots.

Now factor the polynomials using the `factor` command.

Why does the `factor` command not factor them into linear factors?

Now, using `solve`, solve for the roots. Using Maple, verify that the 4 roots of the polynomial g are correct.

Now, using `fsolve`, solve for numerical approximations of the roots, including the complex roots.

Read the help page for `?fsolve` to find out how to get the complex roots.

Solution 3

Question 4

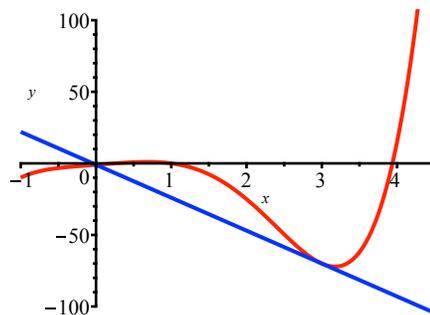
Shown in the plot below is a plot of $f := x^5 - 4x^4 + 4x - 1$ on the domain $-1 < x < 4.5$. Shown also is the line tangent to $f(x)$ at $x = 3$.

Use Maple to reproduce the plot. Do this by using the `diff` and `eval` commands to compute the slope of $f(x)$ at $x = 3$ so you can determine the equation for the tangent line.

```
> f := x^5-4*x^4+4*x-1;
```

$$f := x^5 - 4x^4 + 4x - 1$$

(7.1)



Now express the area enclosed by the tangent line and $f(x)$ as a definite integral. Evaluate the definite integral using Maple.

► Solution 4

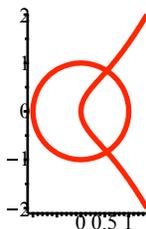
▼ Question 5

A car is travelling at velocity $v(t) = 30 \cdot t \cdot (4 - t)$ *kmph*.
 What is the maximum velocity on $0 \leq t \leq 4$?
 How far does the car travel on $0 \leq t \leq 4$?

► Solution 5

▼ Question 6

Consider the two equations $y^2 = x^3 + x$ and $x^2 + y^2 = 1$ which are the equations of an elliptic curve and a circle. Graph the two curves together using the `implicitplot` command in the `plots` package. Improve the smoothness of the plot using the `grid` option. The circle will appear distorted. Find out from `?plot,options` how to scale the axes so that the circle looks like a circle (so that the plot looks like the figure below).



You can read off from the plot roughly where the curves intersect. Use Maple to calculate the intersection points accurately. Do this by solving two simultaneous equations.

► Solution 6

└ The last four questions are to get you to write some loops and to experiment.

▼ Question 7

└ Suppose you are given a Maple list R of numbers which are the roots of a polynomial $f(x)$. Write a Maple for loop that constructs the polynomial. For example, if $R = [1, 3, 5, 11, 12]$ then the value of f should be the polynomial $(x - 1)(x - 3)(x - 5)(x - 11)(x - 12)$.

► Solution 7

▼ Question 8

└ If $\int_a^b f(x) \cdot g(x) dx = 0$ we say $f(x)$ and $g(x)$ are orthogonal on $[a, b]$. Show that $\sin(n \cdot x)$ and $\cos(m \cdot x)$ are orthogonal on $[-\pi, \pi]$ for all $1 \leq n \leq 5, 1 \leq m \leq 5, n \neq m$. Use a double for loop. To force Maple to print something use the **print(...)** command.

► Solution 8

▼ Question 9

└ Consider the odd primes 3, 5, 7, 11, 13, 17, 19, etc.

Let S be the odd primes congruent to 1 mod 4 and T be the odd primes congruent to 3 mod 4.

So $S = 5, 13, 17, 29, \text{etc.}$ and $T = 3, 7, 11, 19, 23, \text{etc.}$

Suppose we go through the odd primes in order counting the number of primes in S and T.

So at the first step $S = \{ \}$ and $T = \{3\}$ so T has more primes than S.

At the second step $S = \{5\}$ and $T = \{3\}$ so S and T have the same number of primes.

At the third step $S = \{5\}$ and $T = \{3,7\}$ so T has more primes than S again.

Here is a little table counting the size of S and T for the first 10 primes

Prime	3	5	7	11	13	17	19	23	29	31
S	0	1	1	1	2	3	3	3	4	4
T	1	1	2	3	3	3	4	5	5	6

└ Notice that $|T| \geq |S|$ for the first 10 primes. Does it ever happen that $|S| \geq |T|$? Yes, it does. When? How often?

Using the builtin **nextprime** command write a loop that counts $|S|$ and $|T|$ for primes up to 10^6 and prints out the the first time $|S| > |T|$ and, at the end, how often $|S| > |T|$.

► Solution 9

▼ Question 10

Let $A = \int_a^b f(x) \, dx$. Recall that the value of A may be approximated by the **Trapezoidal rule** on n intervals of width $h = \frac{(b - a)}{n}$ using the formula

$$T_n = \frac{h}{2} \cdot (f(a) + 2 \cdot f(a + h) + 2 \cdot f(a + 2 \cdot h) + \dots + 2 \cdot f(a + (n - 1) \cdot h) + f(b)) .$$

For $f(x) = x^2 \cdot \sin(2 \cdot x)$ and $a = 0, b = 1$, calculate A using Maple to 10 decimal places by using Maple's integration command. Now using a Maple loop, calculate T_8, T_{16} and T_{32} using 10 digit arithmetic (the default) .

► Solution 10