MATH 158 Assignment 5, Spring 2013

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Due **Wednesday** March 27th at 4:20 pm.

13.1 Probability Distributions of Continuous Random Variables

Exercises 1, 13, 20, 26, 43 and
If the average waiting time to see a doctor at VGH is 2 hours, what is the probability that you will wait less than one hour? More than 4 hours? [Assume that the waiting time is exponentially distributed.]

13.2 Expected Value and Standard Deviation

[Note, the solutions for exercises 15, 17, ..., 27 are out of order.]

Exercises 2, 8, 13, 20, 28.
For question 20, calculate also the average and sketch the graph of $f(x)$ showing the median and the average. Is the median less than or greater than the average?

13.3 The Normal Distribution

Exercises 1, 5, 10, 20, 21, 22.

11.3 Applications of Differential Equations

11.3 Exercises 5 and 7 plus the following exercise.

Exercise 17 of 11.3 used $P'(t) = kP(t) + I$ to model population growth of a country with constant (net) migration I. Here $P(t)$ is the population at time t and k is the natural growth rate. In exercise 18, using data for Canada from 1990, you solved the DE for $P(0) = 22$ million, $k = 0.8\%$ per year, and $I = 0.2$ million per year.

But $k = 0.8\%$ is not accurate anymore. Using google, I found out that currently, each woman in Canada has on average 1.50 babies in her lifetime (this has remained almost constant over the last 10 years) and that the average age of a woman giving birth is now 29.6 years (this has been increasing slowly). This means that the population under 50 years of age is now in decline; in 29.6 years, the population under 50, in the absence of migration, will shrink by 25%. Using this data one can deduce that the natural growth rate for Canada is now $k = -0.00972$.

Given the current population of Canada is close to 35 million and current net migration is about $I = 0.2$ million per year, using $k = -0.00972$, solve $P'(t) = kP(t) + I$ for $P(t)$. If k and I do not change, what would the long term population in Canada be? If k does not change, what net migration rate I is needed to offset the natural decline in the population? [Hint: if $P(0) = 35$ million, and $k = -0.00972$, what value of I is needed so that $P'(t) = 0$?]