MATH 340 Assignment 5, Fall 2007

Michael Monagan

This assignment is due Friday November 2nd at the beginning of class.
For problems involving Maple please submit a printout of a Maple worksheet.
Late penalty: -20% for up to 24 hours late. Zero for more than 24 hours late.

Polynomial Interpolation

1 Interpolate the data points $(1,1),(2,-1),(0,2)$ in $\mathbb{Z}_7^2$ using
(i) Newton interpolation and (ii) using Lagrange interpolation.

2 Let F be a field and let $x_1 \neq x_2 \neq \ldots \neq x_n \in F$.
Prove that the n Lagrange polynomials

$$L_i = \frac{\Pi_{j=1}^n (x - x_j)}{x - x_i}, \quad 1 \leq i \leq n$$

are linearly independent in $F[x]$.

Section 2.5: Irreducible Polynomials

Exercises 1, 2, 6, 9, 10.

Do questions 1, 2 and 9 by hand. Use Maple to answer question 10.

Complex numbers

1. Let $i = \sqrt{-1}$, $a = (2+3i)$ and $b = (1-2i)$. Calculate $a+b$, $a\,b$ and a/b.
2. Convert $a = 2 - 2i$ and $b = 2i$ to polar co-ordinates and calculate a^2, ab and a/b in polar form. By hand, draw the points a, b, a^2, ab and a/b in the complex plane.
3. Let $z_1 = r(\cos\theta + i\sin\theta)$ and $z_2 = s(\cos\omega + i\sin\omega)$.
Show that $z_1/z_2 = r/s\,[\cos(\theta-\omega) + i\sin(\theta-\omega)]$.
4. Let $\mathbb{Z}[i] = \{a + bi : a, b \in \mathbb{Z} \text{ and } i^2 = -1\}$ and let addition and multiplicaton in $\mathbb{Z}[i]$ be defined as for $\mathbb{C}$. The set $\mathbb{Z}[i]$ is called the set of Gaussian integers. Prove that $\mathbb{Z}[i]$ is a subring of $\mathbb{C}$. See Lemma 2.2.4.

The Fundamental Theorem of Algebra

1. By hand, calculate the real and complex eigenvalues AND eigenvectors of the matrix

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Using Maple, factor the polynomials $x^4 - 3x^2 + 1$ and $x^4 + x^2 + 1$ over $\mathbb{Q}$. Then determine all real and complex roots by applying the quadratic formula by hand.

3. Determine all real and complex roots of the polynomials $x^4 + 3x^2 + 1$ and $x^5 - 1$ in Maple using both the `solve` and `fsolve` commands.
   Observe that $f(a + bi) = 0 \implies f(a - bi) = 0$.