MATH 340 Assignment 4, Fall 2017

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This assignment is due Friday October the 20th at 11:20 am.
Late penalty: -20% for up to 72 hours late. Zero after that.

Section 2.2: Subrings and Subfields

Find a subring of $\mathbb{Z}_8$ with 4 elements.
Is it a subfield? Justify your answer.

Section 2.3: Review of Vector Spaces

Exercises 1, 9, 12.

Let $M_2(\mathbb{Z}_2) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \ : \ a, b, c, d \in \mathbb{Z}_2 \right\}$ and let $\mathbb{Z}_2^4 = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : a, b, c, d \in \mathbb{Z}_2 \right\}$.

So $M_2(\mathbb{Z}_2)$ is the set of 2 by 2 matrices with entries in $\mathbb{Z}_2$ and $\mathbb{Z}_2^4$ is the set of vectors of dimension 4 over $\mathbb{Z}_2$. Show that the vectors spaces $M_2(\mathbb{Z}_2)$ and $\mathbb{Z}_2^4$ are isomorphic.

Section 2.4: Polynomials

Exercises 1, 3, 12, 13, 14.

For 12, 13 and 14 do parts (i) and (ii) only.
For 14 follow the tabular method in Example 1.3.7.

Section 2.5: Polynomial Evaluation and Interpolation

Exercises 1, 2, 3, 6, 7, 8.

Lemma 2.5.1 (i) says if R is an integral domain and $f(x) \in R[x]$ and $a \in R$ then (i) $f(a) = 0 \iff (x - a) | f(x)$.
Prove that this is also true for any commutative ring R with identity 1_R.