MATH 340 Bonus 3, Fall 2017

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This bonus question is to be handed in to me by Wednesday November 29th by 11:30am. It is worth a 1% bonus to your grade.

Let p be a prime and $f, g \in \mathbb{Z}_p[x]$ be irreducible over $\mathbb{Z}_p$ with $\deg f = n > 0$.
Let $F = \mathbb{Z}_p[y]/f(y)$ and $G = \mathbb{Z}_p[z]/g(z)$ so that F and G are finite fields with p^n elements.
Let $\beta \in G$ such that $f(\beta) = 0$.
Let $[a] = [a_0 + a_1 y + \cdots + a_{n-1} y^{n-1}]$ be in F and $\phi : F \to G$ be defined by

$$\phi([a]) = [a_0 + a_1 \beta + \cdots + a_{n-1} \beta^{n-1}] = [a(\beta)].$$

Theorem 2.13.3 on page 163 says ϕ is an isomorphism. For $[a], [b] \in F$ we proved in class that $\phi([a] + [b]) = \phi([a]) + \phi([b])$ and $\phi([a][b]) = \phi([a])\phi([b])$. To complete the proof that ϕ is an isomorphism prove that ϕ is bijective.