The Division Algorithm for $k[x_1, x_2, \ldots, x_n]$.

Input $\prec$ a monomial ordering on $\mathbb{Z}^n_{\geq 0}$
$f \in k[X]$, divisors $f_1, f_2, \ldots, f_s \in k[X]$
where $X = x_1, x_2, \ldots, x_s$, $f_i \neq 0$

Outputs quotients $a_1, a_2, \ldots, a_s \in k[X]$ and
remainder $r \in k[X]$ satisfying
(i) $f = a_1 f_1 + a_2 f_2 + \ldots a_s f_s + r$,
(ii) $LT(f_i)$ does not divide any term in r and
(iii) $LM(f) \geq LM(a_i f_i)$ for $1 \leq i \leq s$.

$(a_1, a_2, \ldots, a_s) \leftarrow (0, 0, \ldots, 0)$

$(r, p) \leftarrow (0, f)$

while $p \neq 0$ **do**

find the first i such that $LT(f_i) | LT(p)$.

if $\nexists i$ **then** $(r, p) \leftarrow (r + LT(p), p - LT(p))$ **CASE 1**

else $t \leftarrow LT(p)/LT(f_i)$

$(a_i, p) \leftarrow (a_i + t, p - t f_i)$ **CASE 2**

end if

end while

output $(a_1, a_2, \ldots, a_s, r)$